AN INQUIRY INTO THE PREDICTION OF SECONDARY-SCHOOL SUCCESS

W. G. EMMETT

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There have been many efforts to determine the value of tests in predicting academic or vocational success. The results of most of these studies are dubious because of the operation of selection. . . . . . The research reported by Emmett is unique in that techniques are applied which yield estimates of the regression coefficients, their standard errors, and the correlations which would be obtained had selection not operated. . . . . . This scholarly little monograph should be "required reading" for all persons engaged in attempts to use regression equations in the prediction of academic or vocational success.

Max D. Engelhart in *Psychometrica.*

AN INQUIRY INTO THE PREDICTION OF SECONDARY-SCHOOL SUCCESS

by

W. G. EMMETT

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PREFACE

Mr. W. G. Emmett has been away on national service since the spring of 1940 and has been therefore unable to see this little book through the press himself. It has been a pleasure to his colleagues at Moray House to do this for him, and they accept all responsibility for any misprints overlooked, or the like. As the whole description of the inquiry is necessarily and advisedly somewhat long and detailed, I have thought it wise to give the following short summary in this preface, to assist the reader.

The 765 children who were the subjects of the inquiry were the survivors of a larger number of entrants, who had remained for three years in certain West Riding secondary schools, and included both special-placers and fee-payers, children who had been handicapped by illness or other special influence being alone excluded. They were assessed by the schools after two and again after three years, and these assessments were correlated with the examinations which all had sat before entrance. The data included—

$$\begin{array}{l} x_1 = \text{Rank in secondary-school assessment.} \\ \left.\begin{array}{l} x_2 = \text{Intelligence quotient} \\ x_3 = \text{English score} \\ x_4 = \text{Arithmetic score} \end{array}\right\} \text{at examination before entrance.} \end{array}$$

The candidates at the entrance examinations were all drawn from one year-group. The intelligence quotient was found from a Moray House test, lasting 40 minutes. The English and Arithmetic examinations, lasting 65 and 55 minutes respectively, were set by, and marked under the guidance of, two experienced examiners whose methods have been much influenced by new type forms of test, and one of whom is a competent practical statistician.

The statistical difficulties of combining data from several schools were met by methods which are explained in the article. The correlations of the secondary-school assessment with the three parts of the entrance examination are shown in this table (not the values in brackets, which are explained below):

Assessment with		After 2 years	After 3 years
I.Q.	r_{12}	·442 (·719)	·454 (·736)
English	r_{13}	·323 (·683)	·335 (·702)
Arithmetic	r_{14}	·304 (·671)	·330 (·694)

These correlations, being in a very highly selected group with shrunken standard deviations, are considerably smaller than correlations would be in the whole population of children from whom these 765 are drawn. To compare different experiments and to judge the actual closeness of association it is necessary to correct them to the values the correlations would have in the complete population, for which purpose we require the standard deviations and the intercorrelations of x_2, x_3 and x_4, the three parts of the entrance examination, both in the whole population and in this sample of 765. These are given in the following table:

	In the sample	In the population
σ_2	9·67	15·00
σ_3	12·77	23·44
σ_4	16·01	25·89
r_{23}	·437	·833
r_{24}	·357	·809
r_{34}	·194	·769

The correlations which secondary-school assessment *would have* with x_2, x_3 and x_4 if the whole population of children had been thus assessed can then be calculated by Karl Pearson's formulæ, preferably in Aitken's matrix formulation, and are given in brackets in the table already printed earlier. The smaller coefficients, those not in brackets, are the actual correlations found in this sample of 765 highly selected children. They indicate the extent to which changes in the order of merit of these 765 children take place. But in deciding whether the entrance examination has in a general sense chosen the right children to enjoy secondary-school education it is the larger correlations, those in the brackets, which should be used, which refer to the whole population of children of which those 765 are a sample. In a similar way the order of merit among First League football clubs may, if they are well chosen and equally matched, change violently during a season, and yet perhaps most of the First League teams could beat any Second or Third League team. Correlations among the First Leaguers would be low, those among *all* teams high, and it would be the latter which would indicate that the right teams were in the First League.

Another reason for using the correlations as estimated in the whole population in comparing the parts of the entrance examination is that since English and Arithmetic were in the years in question in the West Riding the dominating grounds of selection (the intelligence test being only used near the border-line), their standard deviations are somewhat more shrunken than that of the intelligence test. (The ratios of shrinkage are in fact ·645 for I.Q., ·545 for English, ·618 for Arithmetic.) In consequence of this the correlation coefficients in the sample will, to a slight degree, unduly favour the intelligence test ; but this bias disappears in the estimated correlations for the whole population, in which it will be seen that the I.Q. has only a small lead on English or Arithmetic, which are equal. The multiple correlation of the three together, with the later school assessment is ·743 after two years, ·764 after three.

It is hoped that the appendices will be of practical value as a guide to workers wishing to make statistical computations like those used in this inquiry. Appendix 1 describes the calculation of regression coefficients *and their standard errors*. Appendix 2, on the influence of selection on the variance and covariances of the criterion, explains in detail, using the data of the present inquiry, Aitken's method of calculation of Karl Pearson's selection formulæ. The use of this technique in all follow-up inquiries would enable comparisons to be made independent of the shrinkage of range, due to selection, in the secondary schools. It is hoped that the present inquiry may form a model in this respect. Appendix 3 describes the calculation of the reciprocal of a matrix by Aitken's method.

GODFREY H. THOMSON.

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AN INQUIRY INTO THE PREDICTION OF SECONDARY-SCHOOL SUCCESS

1. Introduction

In 1935 an investigation was undertaken in the West Riding of Yorkshire by the Education Officer, Dr. J. H. Hallam, to find out whether the scholarship examination for Special Places in secondary schools selected those children best fitted for secondary education. The co-operation of the Staff of the Education Department of the University of Edinburgh was enlisted in planning the procedure and in carrying out the statistical analysis of the resulting data. The preliminary results of the inquiry relating to the performance of entrants in 1933 after two years at the secondary school were reported by Professor Godfrey H. Thomson in 1936.[1] Since that time the statistics for these children after three school years, and for another year's entry after two and three years, have come along.

The present report combines the data collected at the different times. It is naturally the result of the collaboration of many people: the Education Officer of the West Riding, Dr. J. H. Hallam, and his successor, Mr. A. L. Binns, the staff of the Education Authority, and the head masters and head mistresses of the secondary schools; and in Edinburgh, Professor Godfrey H. Thomson, Messrs. A. G. Rodger and A. S. Mowat.

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[1] Godfrey H. Thomson, "The value of intelligence tests in an examination for selecting pupils for secondary education," *B. J. Ed. Psych.*, 1936, **16**, 174.

The present writer is only responsible for the statistics of the later returns and for their combination with the earlier returns. Professor Thomson and Mr. D. N. Lawley have been constantly consulted over the statistical treatment, and many thanks are due to them for their kind assistance. Thanks are also due to the Education Committee of the West Riding for their permission to publish the report.

2. Outline of Experimental and Statistical Procedure

Scholarships to secondary schools were awarded on the results of a group verbal intelligence test, an English paper and an Arithmetic paper. After the successful candidates together with fee-payers had attended the secondary school for two years, they were arranged in order of merit by their head teachers; a further order of merit was drawn up after three years at the school. This procedure was followed for entrants in 1933 and 1934.

The scores in each of the three parts of the entrance examination and the orders of merit were then correlated separately for each school and for each year's entry.✓ Corresponding correlation coefficients were then formed into weighted averages, giving final grids of coefficients showing the interrelation of the four variables. From these grids regression coefficients and multiple and partial correlations were calculated. The regression coefficients show the relative effectiveness of the three parts of the examination in predicting the criterion of the head teachers' assessments of the pupils. The multiple correlation gives the highest attainable correlation of the examination as a whole with the criterion, each section of the examination receiving a weight proportional to its regression coefficient. The partial correlations show the extent to which each section of the examination measures in the criterion what the other two sections do not themselves measure.

3. The Examination at Entrance to Secondary Schools

Each of the examinations in 1933 and 1934 was held on a Saturday morning in the middle of February and was open to children of age between 10:0 and 11:11 on March 31st of the examination year. Candidates were selected by a preliminary internal test in English and Arithmetic set by the head teachers of the elementary schools. Of the two-year field of children numbering 50,412 in 1933 and 47,312 in 1934, 26·3 per cent. and 26·8 per cent. respectively took the examination. In 1933 2·5 per cent. of the field failed to sit, after recommendation as candidates: in 1934 the figure was 3·0 per cent. The papers in each of the two years were, as stated above, an intelligence test, an English paper and an Arithmetic paper; questions in each subject were of closely similar type in both examinations.

The intelligence test, Moray House Test No. 17 in 1933 and No. 19 in 1934, was constructed by the staff of the Education Department of the University of Edinburgh. It contained 100 questions of well-known types, such as analogies, classification, synonyms, antonyms, number series, proverbs and short problems. Answers were for the most part given by underlining the correct response out of several alternatives: writing was reduced to a minimum. A period of 40 minutes was allowed for working, there being no internal timing. The test was preceded by a practice test of eight questions, for which ten minutes were allowed;

the practice test was not marked. Norms were based on the scores of a background group of about 1,000 children aged eight to fourteen years, together with the scores of a random sample of about 1,000 candidates. Raw scores were converted into intelligence quotients based on a mean quotient of 100 with standard deviation of 15 for an unselected sample of population. The I.Q. thus obtained may be regarded as a derived score which gives due allowance for age at all levels of ability.

The English and Arithmetic papers were set by local examiners of wide experience under the auspices of a central examinations council. Marking was done by a team of about fifty assistant examiners, whose marks were adjusted to a common standard by the chief examiners. Separate papers in each subject were set for the 10 + children, " Juniors " so-called, and for the 11 + or " Seniors."

The English paper was of the usual type, the time allowed being 65 minutes ; it contained essay questions, requiring about half the allotted time. The Arithmetic paper occupied 55 minutes : its first part contained questions in which marks were given for accuracy of working, while the second part was marked chiefly for good method and clear thinking. An age allowance of 1·2 per cent. per month of the maximum possible mark of 200 in the two subjects was given.

Percentage frequencies for each score interval of ten points are given for the several papers in Table 1 : these data are based on representative groups of candidates, each numbering about 1,000.

The mean raw scores given in Table 1 show that each paper taken as a whole was of a suitable order of difficulty. The scores in the two intelligence tests have a much smaller spread, as measured by the

TABLE 1

SHOWING PERCENTAGE FREQUENCIES OF RAW SCORES OF CANDIDATES IN THE ADMISSION EXAMINATIONS

(1) Scores are uncorrected for differences in age.

(2) The maximum possible score in each paper was 100.

(3) The scores are those of children from schools judged to be representative of the West Riding.

(4) No intelligence test scores were available for the samples of Senior candidates.

Score Interval	1933 EXAMINATION					1934 EXAMINATION				
	Juniors (aged 10+)			Seniors (aged 11+)		Juniors (aged 10+)			Seniors (aged 11+)	
	Intell. test M.H.T. 17	Engl. paper	Arith. paper	Engl. paper	Arith. paper	Intell. test M.H.T. 19	Engl. paper	Arith. paper	Engl. paper	Arith. paper
	%	%	%	%	%	%	%	%	%	%
90–100	0·0	1·0	1·9	0·2	1·4	0·0	0·5	2·4	0·8	1·4
80– 89	0·3	2·5	4·4	2·5	2·7	1·3	3·3	4·7	3·6	5·1
70– 79	4·0	8·5	8·3	7·5	8·3	5·3	9·0	9·1	9·6	7·3
60– 69	11·4	15·8	12·1	16·9	12·8	12·5	14·8	11·7	15·4	11·4
50– 59	28·8	19·7	18·3	19·9	15·5	24·4	18·9	14·7	18·9	15·2
40– 49	31·1	19·7	14·9	20·4	18·4	29·3	19·6	17·4	19·2	15·8
30– 39	20·4	17·5	14·1	19·2	16·2	19·6	17·4	14·9	16·6	16·5
20– 29	3·4	8·8	14·7	8·3	11·9	5·9	10·4	11·2	9·7	11·7
10– 19	0·6	5·4	7·4	4·0	8·0	1·4	4·5	8·3	4·6	9·0
0– 9	0·0	1·1	3·9	1·1	4·8	0·3	1·6	5·6	1·6	6·6
Number in sample	943	943	943	1064	1064	1006	1006	1006	994	994
Mean raw score	48·08	48·12	45·77	47·97	44·57	47·88	47·82	45·68	48·56	43·93
Standard deviation	11·58	17·91	21·24	16·89	20·59	13·54	18·17	22·18	18·50	22·03

standard deviation, than those of English and Arithmetic; that is to say, the intelligence tests have a smaller power of discrimination amongst the candidates. They obviously contain some questions too easy and some too difficult for the 10 + candidates. The tests were constructed primarily to differentiate over the whole range of ability of 11 + children; very few children in the selected group of candidates may therefore be expected to make low scores, while few children may also be expected to make high scores on account of their youth. It should be noted, however, that since 1934 the technique of test construction at Moray House has been considerably improved, and a recent test would give a much higher standard deviation of raw score from a similar sample of children.

The first awards of scholarships were made to those children with highest scores in the English and Arithmetic papers only. For the border-line group the results of the intelligence test were also taken into account.[1] The method of giving awards is, however, irrelevant to our present purpose; we are not concerned with how the children managed to get into a secondary school, whether by scholarship or payment of fees. We simply correlate their scores in the admission examination with estimates of their progress in the secondary school.

4. The Children in the Inquiry

In the first instance returns were received in two successive years from ten secondary schools showing the orders of merit of the 1933 and 1934 entries after two years' schooling. Children who had been handicapped by illness or other adverse circumstances were excluded from the survey. Later, similar returns came in from these schools with orders of merit after three years' schooling. The numbers of children in the latter returns were naturally less than those in the former on account of wastage; in some schools the numbers had dwindled so much that it was not considered worth while to compute statistics for them.

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[1] In 1933 and 1934, the intelligence test was used as an adjunct to the English and Arithmetic papers by way of experiment. In 1935 and later years the intelligence quotients were added to the English and Arithmetic scores, each equally weighted, for the determination of all awards.

We thus had two series of statistics, one for assessments after two years, and one for assessments after three years, relating to different numbers of children, as follows :

	Assessed after 2 years	Assessed after 3 years
1933 Entries	613	421
1934 Entries	473	344
Total ..	1086	765

It was then decided to recalculate all the statistics for assessments after two years in respect of precisely those children, 765 in number, who were assessed after three years. In this way we obtained two sets of correlation coefficients which were equally influenced by selection and were therefore strictly comparable. This article relates only to the data from these 765 children.

Table 2 shows the distribution of the intelligence quotients of the children under review. Table 3 shows the distribution of the children amongst the schools, according to year of entry and age group, 10 + or 11 +. The standard deviation of I.Q. shown in Table 2 is greater than that in Table 3, since the

TABLE 2

SHOWING THE DISTRIBUTION OF INTELLIGENCE QUOTIENTS OF THE CHILDREN IN THE INQUIRY

Intelligence Quotient	85-89	90-94	95-99	100-104	105-109	110-114	115-119	120-124	125-129	130-134	135-139	140-144	145-149	150-154	Total
Number of children	1	2	4	12	27	46	77	149	162	120	101	48	12	4	765

Mean I.Q. = 126.21. Standard deviation of I.Q. = 10.02.
The standard deviation has been corrected for grouping errors.

TABLE 3, Showing

(1) The schools under survey.
(2) The number (N) of children in each school.
(3) The mean I.Q. of such children.
(4) The standard deviation of their I.Q.'s about the school mean.
(5) The correlation between the head teacher's order of merit after two years and the order of merit after three years.

School Identification No.	JUNIORS (10-year-olds)				SENIORS (11-year-olds)			
	N	Mean I.Q.	$\sigma_{I.Q.}$	Correlation 2 yrs./3 years	N	Mean I.Q.	$\sigma_{I.Q.}$	Correlation 2 yrs./3 yrs.
Entries, 1933								
2	18	120·9	10·37	·93	22	117·9	14·68	·86
3	25	129·2	6·92	·55	48	121·7	9·97	·64
4	36	127·3	10·55	·84	44	123·8	8·83	·66
5	33	124·3	10·46	·88	—	—	—	—
6	36	126·6	10·10	·77	48	122·3	9·54	·77
9	27	128·8	10·74	·87	20	125·8	9·18	·70
10	38	130·2	9·83	·76	26	122·0	12·25	·72
Totals and means	213	127·09	9·88	·818	208	122·31	10·38	·721
Entries, 1934								
2	—	—	—	—	28	128·1	9·48	·86
3	36	129·5	10·92	·65	34	127·0	6·86	·45
4	38	127·0	11·62	·67	32	128·1	10·32	·74
6	53	129·5	8·80	·71	47	126·4	7·84	·78
10	37	127·5	7·36	·75	39	129·3	9·03	·83
Totals and means	164	128·47	9·62	·699	180	127·71	8·57	·758

Total number of children = 765
Means of all values in table. I.Q. = 126·21
$\sigma_{I.Q.}$ = 9·67
Correlation Coefficient = 0·755

Notes

(1) The means of the standard deviations have been obtained by taking the square root of the averaged variances and applying Sheppard's correction for grouping.

(2) The average of the correlation coefficients is a weighted average obtained by Fisher's z technique.

squared deviations in the former are taken about the general mean, while in the latter they are taken about the means of the respective schools.

Boys formed 60·1 per cent. of the 421 entries in 1933, and 63·4 per cent. of the 344 entries in 1934. The Seniors or 11 + children include those who failed to gain scholarships as ten-year-olds; they comprise roughly 60 per cent. of the 11 + candidates. Of the Juniors 11 per cent. were fee-payers with mean I.Q. of 116·3. Excluding these fee-payers the Juniors had a mean I.Q. of 129·29, and the Seniors, 126·97, with a mean for the combined groups (still excluding fee-payers) of 128·17.

A rough check on this mean can be obtained from the percentage of children in the total field receiving awards, which was about 8·5 per cent. in 1933 and 1934. If we assume that all children offered scholarships proceeded to the secondary school, and that the English and Arithmetic scores of these children, by which awards were made, correlated perfectly with their I.Q.'s, we should expect this mean I.Q. to be 127·5, this being the mean I.Q. of the top 8.5 per cent. of a normally distributed population with $\sigma_{\text{I.Q.}} = 15$. The agreement between the roughly estimated mean I.Q. of 127·5 and the ascertained value of 128·17 is satisfactory, and substantiates the standardisation of the intelligence tests.

5. The Criterion of the Head Teacher's Order of Merit

At the beginning of the investigation the head teachers of the secondary schools concerned were asked to rank in order of merit those children who entered their schools in 1933 and 1934, "having regard to their performance in the normal subjects of the school curriculum," and avoiding "ties" as far as possible. They were also requested to indicate "any particular circumstances which may have had an effect on the progress of any child, such as poverty or lack of encouragement by the parents or lack of proper accommodation to do homework." The directions were thus of a general nature, and it was left to the head teachers to classify the children as best they could in the light of the data available at each school. Reports from the head teachers show that the greatest care was taken in making the assessments in spite of the difficulty in comparing children who were in different classes or were taking different school subjects. It was of importance to ascertain the extent to which the assessment of a child after three years was influenced by that after two years, and we learnt that as far as possible the assessments were made from independent series of marks.

Comments from the several head teachers are given below.

School 2.—" A full staff meeting was called. Every child was discussed in relation to general intelligence, term work, examination results, home conditions and health. The form teachers then compiled an order of merit based on the lists (of marks) and on the comments and information supplied at the staff meeting."

School 3.—" The order has been arrived at by taking the total of examination marks gained in each of the Autumn, Spring and Summer terms of the school year just closed and equating them as far as possible."

School 4.—" The order of merit has been based primarily upon the term order obtained in school examinations, in which all school subjects have counted equally for purposes of marks. . . . This method does not really give any satisfactory comparison of boys in different forms who may even have been pursuing different subjects. I do not think that any valid comparison between such boys can in fact be obtained. Where boys were in different forms, regard was had to my own and my colleagues' knowledge of the boys concerned."

School 5.—" We had staff meetings. . . . Every teacher who taught the children put them in an order of merit, the member's ranks were added up, an average was taken and the children were assessed in accordance with the result. The assessment after three years could not thus possibly be affected by the assessment after two years. I have always felt that there were too many factors to be taken into account for the questionnaire to have real scientific value." (This method of assessment, in which the ranks in different subjects were averaged, is perhaps the best possible, since each subject is thereby given equal weight.)

School 6.—" I have pointed out the wellnigh impossible task of making an order of merit on conditions which vary so considerably. I gave up the attempt of making a mere percentage of marks only, a basis of classification. My first raw list consists of all marks gained throughout the school irrespective of the number of the subjects, given as a percentage. This list presents innumerable anomalies in that the 'two-language' people frequently lag behind the handicraft people. Marks for the handicraft people are generally higher than those for the two-language course. I therefore quite arbitrarily place the language people in a position which they ought in my opinion to occupy. In doubtful cases I consulted my colleagues."

School 9.—" The basis adopted was Form position decided by marks given for tests and examinations only. As the pupils were in different forms it was extremely difficult to allow for the difference in standard of the forms. I was not satisfied that the order of merit was sufficiently correct to be used in a scientific investigation. There is no possibility of the assessment after three years having been affected by the assessment after two years."

School 10.—" I have assessed the children according to the marks they gained in their annual examinations. Consequently the assessment for three years could not be affected in any way by the assessment for two years."

In general, assessments were made by summing marks in the different school subjects. It is unlikely that any adjustments were made to give equal weight to each subject, except in the case of School 5, where the excellent method of averaging the ranks in different subjects was adopted. Age allowances could not very well have been made, but in any case these would have no appreciable influence on the results of the inquiry.

Juniors and Seniors, i.e. the 10 + and 11 + entries, were ranked separately, since they had taken different papers in English and Arithmetic in the admission examination.

Correlation coefficients between the orders of merit after two years and after three years are given in Table 3, page 18. The mean correlation coefficient of 0·755 shows a fairly high consistency of performance from one year to the next, especially for such a highly selected group ; it also indicates that the forebodings of some of the head teachers as to the scientific value of the assessments have not been realised, provided, of course, the two assessments were made on independent data. We may estimate roughly the correlation that would have resulted had the same methods of assessment been applied to a representative sample of the population of children, by making the assumption that intelligence quotients alone were the basis of selection of those remaining at school after three years. Taking the standard deviation of I.Q. as 15 for the population, the figure on which Moray House norms are based, and applying the Otis-Kelley formula, we find the corrected correlation coefficient to be about 0·90, a quite satisfactory figure.

Valentine[1] found a correlation of 0·81 (s.e. = 0·026) between orders of merit for 169 pupils after two and three years in the secondary school. The degree of selection of these pupils is, however, not available.

6. The Intercorrelations

We have four variables, a criterion and the scores in the Intelligence, English and Arithmetic sections of the admission examination. The criterion is two-fold, an assessment by the head teachers after two years at school and after three years. We thus have for each school two sets of six correlation coefficients, one set for each criterion. The intercorrelations of the three independent variables, Intelligence, English and Arithmetic, will be the same for each set. Product moment correlation coefficients were calculated by the method of "diagonal adding," scores being grouped with intervals of 5. Head teachers' ranks were treated as ordinary marks in reversed order; although these ranks are not in normal distribution, the effect of this approximation on the correlation coefficient was found to be negligible, as was to be expected. All correlations were checked by finding the sums of squared deviations from two arbitrary zero points. Sheppard's correction for grouping was not applied, since its effect was found to be but small. As the correction always increases slightly the value of the correlation coefficient, the degree of relationship between the several variables has been underestimated, an error on the safe side. Scores in the English and Arithmetic papers were taken for purposes of correlation without age allowances, since allowances had been awarded on the total score in the two papers and separate allowances for each paper were not available. The error thus introduced was small, for the correlation between age and score in a group of age-range of one year is at the most 0·2 and a total correlation of 0·50 is only depressed to a partial correlation of 0·48 with age constant. The intelligence-test scores, however, made allowances for age in that they were expressed as intelligence quotients. The head teachers' orders of merit were probably drawn up irrespective of differences in age.

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[1] C. W. Valentine, *The Reliability of Examinations*, Univ. of London Press, 1932, p. 87.

The correlation coefficients are given in Table 4.

TABLE 4

Showing the Intercorrelations of the Several Variables

(1) S.R. is the School Rank, or Head Teacher's Order of Merit after two years or three years at school as stated.

(2) Juniors are the ten-year-olds, and Seniors the eleven-year-olds.

(3) The intercorrelations of Intelligence, English and Arithmetic are, of course, the same for the three-year group as for the two-year group.

School Identification No.	No. of pupils	$\sigma_{I.Q.}$	Correlation Coefficients								
			S.R. 2 yrs. and I.Q.	S.R. 2 yrs. and Engl.	S.R. 2 yrs. and Arith.	I.Q. and Engl.	I.Q. and Arith.	Engl. and Arith.	S.R. 3 yrs. and I.Q.	S.R. 3 yrs. and Engl.	S.R. 3 yrs. and Arith.
Entries, 1933, *Juniors*											
2	18	10·37	·49	·49	·57	·58	·50	·77	·57	·60	·58
3	25	6·92	·33	·19	·27	·22	·18	-·07	·44	·05	·14
4	36	10·55	·58	·06	·27	·32	·41	·22	·53	·36	·44
5	33	10·46	·35	·34	·37	·43	·63	·51	·57	·39	·49
6	36	10·10	·36	·20	·23	·31	·14	·01	·45	·15	·23
9	27	10·74	·52	·62	·43	·52	·72	·35	·52	·72	·44
10	38	9·83	·38	·50	·17	·59	·25	·21	·36	·48	·26
Entries, 1933, *Seniors*											
2	22	14·68	·68	·68	·55	·65	·60	·50	·74	·59	·62
3	48	9·97	·26	·14	·47	·53	·46	·21	·35	·25	·39
4	44	8·83	·29	·16	·23	·29	·30	·28	·22	·34	·42
6	48	9·54	·38	·47	·10	·34	·50	·18	·41	·22	·21
9	20	9·18	·37	·32	·46	·57	·68	·47	·47	·36	·57
10	26	12·25	·32	·48	·32	·42	·28	·41	·46	·36	·32
Entries, 1934, *Juniors*											
3	36	10·92	·73	·46	·48	·50	·35	·07	·43	·35	·57
4	38	11·62	·55	·40	·29	·64	·45	·40	·52	·48	·38
6	53	8·80	·41	·27	·43	·37	·14	-·11	·33	·33	-·01
10	37	7·36	·54	·25	·25	·20	·41	-·21	·43	·27	·20
Entries, 1934, *Seniors*											
2	28	9·48	·32	·38	·38	·34	·14	·09	·11	·27	·35
3	34	6·86	·46	·35	·19	·30	·18	-·29	·43	·12	·27
4	32	10·32	·38	-·06	·20	·46	·18	·08	·42	·00	·44
6	47	7·84	·44	·37	·17	·62	·28	·37	·61	·43	·21
10	39	9·03	·52	·17	·12	·35	·14	·20	·64	·27	·13
22 groups	765	9·67	·442	·323	·304	·437	·357	·194	·454	·335	·330
No. of *z* values exceeding the mean by more than 2σ . .			1	1	0	0	1	1	0	1	0
No. of *z* values less than the mean by more than 2σ . .			0	1	0	0	0	3	0	0	1

The averaged coefficients at the bottom of the table were obtained by Fisher's z technique.[1]

The correlations between each pair of variables are scattered over a fairly wide range. This might have been expected, for the entries into different schools in a given year are not truly random samples of those receiving scholarship awards. Some successful children were refused secondary-school education by their parents (about 6 per cent. of the total awards), the distribution of ability in different towns and villages of the West Riding will certainly not be uniform, and the level of ability of those leaving the secondary schools before their second or third year had been completed was not likely to have been the same for each school. It is, however, a matter of interest to see how closely these correlations comply with the assumption that they are derived from random samples of the total secondary-school population. For this purpose the extreme values in each group have been tested for significant divergence from the mean of the group. The method, that of Fisher, is illustrated by the following example from the first column of correlation coefficients. The highest coefficient in this column is 0·73, the significance of whose difference from the mean of 0·442 we desire to test.

N	r	z	$N-3$	$\frac{1}{N-3}$
36 765	·73 ·442	·929 ·475	33 699	·03030 ·00143
		Diffce. = ·454		Sum = ·03173

The z values corresponding to the r's are obtained from Fisher's z table. The figure of 699 for the value of

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[1] R. A. Fisher, *Statistical Methods for Research Workers*, 7th edition, 1938, par. 35, p. 209. The numbering of the pages differs according to the edition, but the paragraph numbering has been preserved.

$(N-3)$ for the mean is the sum of the values of $(N-3)$ for the individual observations, that is $765-66$, since there are 22 observations of r. The variance of the values of z is $\frac{1}{N-3}$, and the sum of the two variances, ·03173, is the variance of the difference of the two z's, ·454. The square root of ·03173 or ·1781 is the standard error of this difference. The difference expressed in standard units, or the significance ratio, is ·454/·1781 or 2·55.

In this way we have found the differences expressed in standard units between the outliers of the 198 correlations of the table and the means of their respective groups. The distribution of these differences is shown below:

Limits of difference in standard units	Differences exceeding the mean	Differences less than the mean
3·10–3·19	1	0
2·70–2·79	1	1
2·50–2·59	2	0
2·40–2·49	0	1
2·30–2·39	0	1
2·10–2·19	1	1
2·00–2·09	0	1
Total	5	5

The incidence of these differences in respect of each pair of variables is shown in the last two rows of Table 4. Strangely enough, of the 22 correlations between English and Arithmetic four show deviations (expressed in terms of z) from the mean of more than 2σ, whereas only one out of 22 should exceed 2σ if the samples were truly random. Although the averaging of significantly different data may not be statistically respectable, the circumstances of the experiment seem to preclude any other course, while the omission of the aberrants would present an incomplete picture of the situation.

7. Regression Coefficients and the Prediction of the Criterion

The averaged correlations give us two matrices from which regression coefficients and multiple and partial correlation coefficients can be calculated :

The following notation has been adopted :

x_1 = Rank in secondary school according to head teacher's assessment.
x_2 = Intelligence quotient
x_3 = English score
x_4 = Arithmetic score
} Data from the admission examination.

The correlation matrices are as follows :

TABLE 5

SHOWING AVERAGED CORRELATION COEFFICIENTS

Assessment after 2 years
($N = 765$)

	x_1	x_2	x_3	x_4
x_1	—	·442	·323	·304
x_2	·442	—	·437	·357
x_3	·323	·437	—	·194
x_4	·304	·357	·194	—

Assessment after 3 years
($N = 765$)

	x_1	x_2	x_3	x_4
x_1	—	·454	·335	·330
x_2	·454	—	·437	·357
x_3	·335	·437	—	·194
x_4	·330	·357	·194	—

The reciprocal of the submatrix for variables x_2, x_3 and x_4, with unities as diagonal elements (it is the same for each of the two matrices) was then found, and regression coefficients and their standard errors were calculated by the method given in Appendix 1.

The results follow in Table 6.

The foregoing regression coefficients are the weights by which the *standardised* scores in the three variables

TABLE 6

Showing Regression Coefficients and their Standard Errors

(1) The significance ratio is the ratio of the statistic in question to its standard error.

(2) P represents the probability that the statistic observed in the sample of 765 cases could have arisen by chance alone on the assumption that it was zero in the population from which the present sample was drawn, viz. the secondary-school population after three years' schooling. For example, the probability that the value of $(b_{12} - b_{13})$ for the two-year assessment could have arisen by chance alone is only ·0063, or 6·3 chances in 1000. We may conclude, therefore that b_{12} is significantly greater than b_{13}.

	Assessment after 2 years				Assessment after 3 years			
	Regression Coefficient	Standard Error	Significance Ratio	P	Regression Coefficient	Standard Error	Significance Ratio	P
$b_{12\cdot 34}$	·3177	·0370	8·59	·0000	·3178	·0365	8·71	·0000
$b_{13\cdot 24}$	·1530	·0352	4·35	·0000	·1602	·0347	4·62	·0000
$b_{14\cdot 23}$	·1609	·0339	4·75	·0000	·1854	·0334	5·55	·0000
$b_{12} - b_{13}$	·1647	·0604	2·73	·0063	·1576	·0596	2·64	·0083
$b_{12} - b_{14}$	·1568	·0573	2·74	·0061	·1324	·0566	2·34	·0192
$b_{14} - b_{13}$	·0079	·0499	0·16	·873	·0252	·0493	0·51	·610

Multiple Correlation $r_{1\cdot 234} = \cdot 489$

Multiple Correlation $r_{1\cdot 234} = \cdot 509$

must be multiplied in order to give the best prediction of secondary-school success; they may therefore be regarded as measures of the relative values of the three variables in predicting success in the sample of children from which the present statistics have been derived. It is important to note that regression coefficients for *raw* scores are not to be taken for this purpose, for they are arbitrary quantities which can be changed according to caprice by changing the unit in which the variable is scored.[1] Scores must therefore be standardised in order to put each variable on the same footing, before any just comparison can be made. It may appear at first sight that when scores have been standardised the efficiency of a test, which discriminates better than another similar test, and has a higher standard deviation of raw scores, is not reflected in its regression coefficient. Such a test, however, will have a higher reliability coefficient and in consequence a higher correlation with the criterion. Although its correlation with the independent variables will also be raised, the net effect of its increase in reliability may be shown to be a higher regression coefficient, positive or negative as the case may be.

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[1] A recent valuable inquiry into the prediction of success in School Certificate subjects by means of an examination at 11 + has unfortunately used regression coefficients for raw scores to assess the merits of the predicting variables. (See T. E. Stubbins, *Brit. Journ. Educ. Psych.*, 1940, 10, 16.)

Regression coefficients for standard scores in one sample of the population may be readily converted into those for another sample in which the standard deviations of the variables differ, by making use of the fact that regression coefficients for raw scores are independent of the distribution of the independent variables in the population.[1]

If $x_1, x_2, x_3, \ldots$ are raw scores measured from the mean, and $w_{12}, w_{13} \ldots$ regression coefficients for raw scores, we have

$$\hat{x}_1 = w_{12}x_2 + w_{13}x_3 + \ldots$$

Standardising the scores

$$\frac{\hat{x}_1}{\sigma_1} = \left(w_{12}\frac{\sigma_2}{\sigma_1}\right)\frac{x_2}{\sigma_2} + \left(w_{13}\frac{\sigma_3}{\sigma_1}\right)\frac{x_3}{\sigma_3} + \ldots$$

Writing z_i for $\frac{x_i}{\sigma_i}$ and b_{1i} for $\left(w_{1i}\frac{\sigma_i}{\sigma_1}\right)$,

$$\hat{z}_1 = b_{12}z_2 + b_{13}z_3 + \ldots$$

$$\text{or } b_{12} = w_{12}\frac{\sigma_2}{\sigma_1},\ b_{13} = w_3\frac{\sigma_3}{\sigma_1}, \text{ etc.}$$

Similarly for another sample with standard deviations $\Sigma_1, \Sigma_2, \Sigma_3$, etc., the regression coefficients are given by

$$B_{12} = w_{12}\frac{\Sigma_2}{\Sigma_1},\ B_{13} = w_{13}\frac{\Sigma_3}{\Sigma_1}, \text{ etc.}$$

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[1] R. A. Fisher, *Statistical Methods for Research Workers*, 7th edn., 1938, par. 25, p. 135.

Hence $B_{12} = b_{12}\frac{\sigma_1}{\sigma_2}\frac{\Sigma_2}{\Sigma_1}$, $B_{13} = b_{13}\frac{\sigma_1}{\sigma_3}\frac{\Sigma_3}{\Sigma_1}$, etc.

The regression equation for the second sample in terms of standardised scores Z_i thus becomes:

$$\hat{Z}_1 = b_{12}\frac{\sigma_1}{\sigma_2}\frac{\Sigma_2}{\Sigma_1} Z_2 + b_{13}\frac{\sigma_1}{\sigma_3}\frac{\Sigma_3}{\Sigma_1} Z_3 + \ldots$$

8. The Influence of Selection

Up to the present our statistics have concerned a sample of secondary-school children of an entirely artificial composition. The sample is the end result of a selection by many diverse influences from the population of elementary-school children of examination age. The first selection from this population was that of candidates for the admission examination by means of a test in English and Arithmetic given in the elementary schools. From these candidates the entrants to the secondary schools were selected mainly on the results of the English and Arithmetic papers of the admission examination, for the border-line candidates also by performance in the intelligence test. Another class of entrants was the fee-payers, about 15 per cent. of the total, admitted in spite of a poor examination performance. Some successful children will have been refused secondary education by their parents, while others will have been excluded by accidents of circumstance such as ill health. Further selection will have taken place by the departure of a number of children, mostly the duller, before the end of their third school year. The situation is thus one of extreme complexity. If, however, those influences other than examination scores which have determined selection are of a random nature and are uncorrelated with the criterion, they will have no bearing on the statistics of the inquiry. We shall necessarily have to ignore the unknown elements in the situation, making the only practicable assumption that these are random, and that selection has occurred solely through the scores of the three parts of the admission examination.

The effect of these last three processes of selection is to reduce the standard deviations of the several variables and their intercorrelations below the values for the unselected population. The standard deviations will not all be reduced to the same relative extent, however; for since English and Arithmetic were the dominating grounds of selection, the fall in standard deviation of the intelligence test will be mostly determined indirectly through its correlation with the two other variables and thus will be proportionately less. As a result the correlation of the intelligence test with the criterion will not suffer so great a reduction as that of English and Arithmetic, and therefore the regression coefficients calculated for the sample will unduly favour the intelligence test. To make a fair comparison, we must calculate a whole new set of statistics which relate to the complete population of children about to leave the elementary school. This is a standard population likely to show but little difference from one educational area to another, so that its statistics will have general application; it is also that population from which the admission examination has to select those suited to secondary education, and its statistics will therefore give the truest interpretation of the results from the admission examination.

We first require to know the standard deviations and intercorrelations of the three sections of the examination in the unselected population. Fortunately we have in our records at Moray House the scores in a similar examination of over 1,100 children of age 10·0 to 10·11 from West Riding schools, chosen to be as nearly representative of the district as possible.[1] The papers were set by the same examiners as those in the present inquiry, and the contents of the two groups of papers were closely parallel.

We have to find next what the standard deviation of the head teachers' assessments would have been, if they had been made on the whole population. This is done by the use of Aitken's matrix formulation of Karl Pearson's selection equations,[2] applied to the case where one variable, the criterion or x_1, is influenced by induced selection through three other variables, x_2, x_3 and x_4. We thus arrive at the statistics given in Table 7, obtaining the correlation coefficients with the

TABLE 7

SHOWING STANDARD DEVIATIONS AND INTERCORRELATIONS FOR THE SECONDARY-SCHOOL SAMPLE AND FOR THE UNSELECTED POPULATION OF SCHOOL CHILDREN

Standard Deviations

	Sample	Population	
		Assessment after 2 years	Assessment after 3 years
School Rank	$\sigma_1 = 11{\cdot}233$	$\Sigma_1 = 14{\cdot}65$	$\Sigma_1 = 14{\cdot}97$
I.Q.	$\sigma_2 = 9{\cdot}67$	$\Sigma_2 = 15{\cdot}00$	$\Sigma_2 = 15{\cdot}00$
English	$\sigma_3 = 12{\cdot}77$	$\Sigma_3 = 23{\cdot}44$	$\Sigma_3 = 23{\cdot}44$
Arithmetic	$\sigma_4 = 16{\cdot}01$	$\Sigma_4 = 25{\cdot}89$	$\Sigma_4 = 25{\cdot}89$

Notes.—(1) The value of σ_1 was found by averaging the variances of the ranks in each school, allowing for the loss of one degree of freedom in making the estimate for each school; it is thus an artefact depending on the number of children in the school. It is, of course, the same for both assessments. Similarly the derived values of Σ_1 are of an artificial nature, but they may be legitimately used for estimating the regression coefficients and correlations in the population.

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[1] The data were acquired in the course of an investigation by the West Yorkshire National Union of Teachers, published by the University of London Press.

[2] A. C. Aitken, *Proc. Edinburgh Math. Soc.*, 1934, 4, 106. An illustrative computation is given in Appendix 2.

(2) Σ_2 is the s.d. of I.Q. in the population, and is taken as 15·00 since Moray House tests are standardised to this scale.

Correlations

The correlations for the selected sample are given in Table 5.

The correlations for the population are as follows:

Assessment after 2 years

	x_1	x_2	x_3	x_4
x_1	—	·719	·683	·671
x_2	·719	—	·833	·809
x_3	·683	·833	—	·769
x_4	·671	·809	·769	—

Multiple Correlation
$R_{1\cdot234} = \cdot743$

Assessment after 3 years

	x_1	x_2	x_3	x_4
x_1	—	·736	·702	·694
x_2	·736	—	·833	·809
x_3	·702	·833	—	·769
x_4	·694	·809	·769	—

Multiple Correlation
$R_{1\cdot234} = \cdot764$

Note.—The standard errors of the correlations with the criterion, x_1, can only be roughly estimated; they are in the neighbourhood of 0·019. The correlations between x_2, x_3 and x_4 have standard errors of about 0·012.

criterion in the population as a by-product. Small letters denote statistics for the selected sample, and large letters for the whole population. All the data are based on observed values, except Σ_1 and correlations with the criterion in the population, which are calculated from the others.

The correlation coefficients relating to the assessment after two years show but little difference from those after three years. Contrary to expectation each variable predicts slightly more accurately after the passage of another year; this difference, however, is not significant. It could have arisen from random influences alone, while a more accurate grading of the children at the second assessment as a result of experience may also have had its effect.

Regression coefficients for standard scores in the population may now be found either from the correlation matrices in Table 7, or more readily from the regression coefficients in the sample by making use of the fact that regression coefficients for raw scores are independent of selective influences (see page 29).

Taking for illustration the regression of assessment after two years on intelligence quotient, we have from Table 6, b_{12} (for standard scores in the selected sample) $= \cdot 3177$. The regression coefficient for raw scores, w_{12}, is $b_{12} \times \frac{\sigma_1}{\sigma_2}$, and that for standard scores in the population is $b_{12} \times \frac{\sigma_1}{\sigma_2} \times \frac{\Sigma_2}{\Sigma_1} = \cdot 3177 \times \frac{11 \cdot 233}{9 \cdot 67} \times \frac{15 \cdot 00}{14 \cdot 65}$ $= \cdot 3779$.

Table 8 gives these regression coefficients.

From the regression coefficients for the population the multiple correlations of Table 7 are found. It will be noted from the columns giving the percentage weights of each variable for best prediction that but little difference is shown between the respective weights for the sample and for the population. What differences exist are almost certainly not significant.

We may note that had the degree of selection, as measured by $\frac{\sigma_i}{\Sigma i}$, been the same in each independent variable, the *relative* values of the regression coefficients

TABLE 8

SHOWING REGRESSION COEFFICIENTS FOR THE SECONDARY-SCHOOL SAMPLE AND FOR THE UNSELECTED POPULATION OF SCHOOL CHILDREN

	Assessment after 2 years		Assessment after 3 years	
	Sample	Population	Sample	Population
$b_{12\cdot 34}$	·3177	·3779	·3178	·3699
$b_{13\cdot 24}$	·1530	·2152	·1602	·2205
$b_{14\cdot 23}$	·1609	·1995	·1854	·2250
Percentage weights	%	%	%	%
x_2 (I.Q.)	50·3	47·7	47·9	45·3
x_3 (English)	24·2	27·1	24·2	27·1
x_4 (Arith.)	25·5	25·2	27·9	27·6

for standard scores would have been the same in both sample and population; for the regression coefficient for raw scores, w_{1i} say, is the same in both cases. Thus b_{12}/b_{13} (for the sample) $= w_{12}/_{13} \times \sigma_2/\sigma_3$, and B_{12}/B_{13} (for the population) $= w_{12}/w_{13} \times \Sigma_2/\Sigma_3$, denoting regression coefficients for the population by B_{1i}. These expressions are obviously equal when $\sigma_2/\Sigma_2 = \sigma_3/\Sigma_3$.

Since in most inquiries similar to the present the degree of selection in each predicting variable will be roughly the same, regression coefficients based on the highly selected sample remaining in the secondary school will give a fairly accurate measure of the *relative* prognostic values of the variables, and if this information suffices the somewhat laborious corrections for selection may be omitted. In the present case we have $\sigma_2/\Sigma_2 = \cdot 645$, $\sigma_3/\Sigma_3 = \cdot 545$ and $\sigma_4/\Sigma_4 = \cdot 618$, and we have seen that even with these differing degrees of selection the relative weights of each variable are not greatly changed. If, however, a measure of the predictive value of the whole examination in the population is required, we must first know the standard deviations and intercorrelations of the independent variables in the population, and then must apply the correction for selection before the desired correlations with the criterion in the population can be found.

The standard errors of the regression coefficients for the population bear a simple relation to those for the sample (see Appendix 1). They are given in Table 9, which may be compared with similar data for the sample given in Table 6, page 28.

TABLE 9

SHOWING STANDARD ERRORS OF REGRESSION COEFFICIENTS FOR STANDARD SCORES IN THE POPULATION

x_1 = School Rank ; x_2 = I.Q. ; x_4 = English ; x_4 = Arithmetic.

	Assessment after 2 years				Assessment after 3 years			
	Regression Coefficient	Standard Error	Significance Ratio	P	Regression Coefficient	Standard Error	Significance Ratio	P
$B_{12\cdot34}$	·3779	·0440	8·59	·0000	·3699	·0425	8·71	·0000
$B_{13\cdot24}$	·2152	·0495	4·35	·0000	·2205	·0477	4·62	·0000
$B_{14\cdot23}$	·1995	·0420	4·75	·0000	·2250	·0405	5·55	·0000
$B_{12} - B_{13}$	·1627	·0783	2·08	·0376	·1494	·0756	1·98	·0478
$B_{12} - B_{14}$	·1784	·0696	2·56	·0105	·1449	·0672	2·16	·0308
$B_{13} - B_{14}$	·0157	·0664	0·24	·810	−·0045	·0641	0·07	·944

See Table 6, page 28, for an interpretation of the values of P.

9. THE RELATIVE PREDICTIVE VALUES OF THE THREE EXAMINATION VARIABLES

In estimating the merits of the Intelligence (x_2), English (x_3) and Arithmetic (x_4) papers in their power to predict later performance (x_1), we must distinguish between the merit of each paper (*a*) when considered independently of the other two, and (*b*) when forming part of a team with the other two. In the former case the square of the correlation with the criterion is a suitable figure of merit, since it is the proportion of the variance of the criterion measured by the test. In the latter case we must take into account the correlations of the predicting variables amongst themselves, for if two of the variables are very highly correlated they measure almost the same aspect of the criterion ; the result is that if one variable predicts the criterion slightly better than the other, it will have a much larger regression coefficient, since its mate has but little left to do. The relative merits of the variables *in a team* must therefore be judged by their regression coefficients.

We will consider each case separately and confine ourselves to the statistics as estimated for the population.

(*a*) The squared correlations with the criterion are :

	Assessment after 2 years	Assessment after 3 years
R^2_{12} . .	·517	·542
R^2_{13} . .	·466	·493
R^2_{14} . .	·450	·482

The intelligence test shows itself superior to the other two, and this superiority is almost significant at the 5 per cent. point as judged from the standard errors of the correlations in Table 7.

It is of interest to compare the correlation between intelligence test and criterion with that of the best weighted combination of English and Arithmetic with the criterion. Thus :

	Assessment after 2 years	Assessment after 3 years
R_{12} . .	·719	·736
$R_{1\cdot34}$ . .	·720	·742

These correlations are, of course, estimates, subject to a standard error in the neighbourhood of ·02, and the very close agreement between R_{12} and $R_{1\cdot34}$ might be disturbed in one direction or the other by another investigation in which larger numbers of children were taken. We may, however, say that on the evidence available the substitution of papers in English and Arithmetic by one intelligence test would be justified in an admission examination in which one or other alternative had to be employed.

(*b*) Turning now to the relative merits of the three tests when employed as a battery, the regression coefficients of Table 9 taken along with their P values show that the intelligence test is significantly superior to the two attainment tests for predicting performance both after two years and after three years in the secondary school. Its effectiveness is nearly equal to that of the other two taken together. The English and Arithmetic papers have about equal prognostic value in the team, and their regression coefficients are not significantly different.

The proportion of the variance of the criterion measured by the team is given by the square of the multiple correlation coefficient, that is ·552 for the assessment after two years, and ·584 for the assessment after three years.

10. Partial Correlation Coefficients

The partial correlations for the population derived from the estimated total correlation coefficients of Table 7 are given in Table 10; the total correlations are also given for sake of comparison.

TABLE 10

Showing the Partial Correlations between the Variables, as Estimated for the Population

Assessment after 2 years

$R_{12} = \cdot 719$ $R_{13} = \cdot 683$ $R_{14} = \cdot 671$ $R_{23\cdot 4} = \cdot 561$
$R_{12\cdot 3} = \cdot 371$ $R_{13\cdot 2} = \cdot 219$ $R_{14\cdot 2} = \cdot 219$ $R_{24\cdot 3} = \cdot 476$
$R_{12\cdot 4} = \cdot 404$ $R_{13\cdot 4} = \cdot 352$ $R_{14\cdot 3} = \cdot 312$ $R_{34\cdot 2} = \cdot 292$
$R_{12\cdot 34} = \cdot 267$ $R_{13\cdot 24} = \cdot 166$ $R_{14\cdot 23} = \cdot 166$

Assessment after 3 years

$R_{12} = \cdot 736$ $R_{13} = \cdot 702.$ $R_{14} = \cdot 694$ $R_{23\cdot 4} = \cdot 561$
$R_{12\cdot 3} = \cdot 384$ $R_{13\cdot 2} = \cdot 237$ $R_{14\cdot 2} = \cdot 248$ $R_{24\cdot 3} = \cdot 476$
$R_{12\cdot 4} = \cdot 413$ $R_{13\cdot 4} = \cdot 366$ $R_{14\cdot 3} = \cdot 339$ $R_{34\cdot 2} = \cdot 292$
$R_{12\cdot 34} = \cdot 269$ $R_{13\cdot 24} = \cdot 178$ $R_{14\cdot 23} = \cdot 192$

There is but little difference between the corresponding partial correlations for the assessments after two and three years. The progressive fall in correlation with the criterion (x_1) as first one and then two variables are eliminated indicates that each of the three sections of the entrance examination measures in the criterion in large part what the other two measure. The fall is much the greatest when the first variable is eliminated; this is a consequence of the high correlation between the removed variable and the criterion. The removal of a second variable causes a relatively small fall, since this variable correlates highly with that first removed.

That portion of the criterion measured by one variable and not by the other two is revealed by the partial correlations in the last row of the two tabulations. The intelligence test obviously stands up best to the successive refining processes ; it also has more in common with the English and Arithmetic papers taken separately, than have the latter with each other after eliminating their connection through intelligence, as is shown by the last column of the table.

The standard errors of these partial correlations can only be roughly gauged, since the total correlations themselves were estimated by the selection formula, and their own standard errors are not accurately known. However, the partial correlations are not likely to have a standard error exceeding 0·03.

11. Discussion of Results

The multiple correlation, ·764, representing the degree of concordance between entrance-examination results and performance in secondary school after three years, though as high as might be expected, shows considerable departure from perfection of prediction ; it is therefore of interest to inquire how far prediction might be improved. In the first place we must recognise that there is no consensus about the meaning of success in secondary-school life. Do we mean satisfactory academic attainments, and if so, are we to count as successful a boy who is brilliant in one subject, say Mathematics, and a dullard in most others ? Do we mean qualities of character, which are developed by the social contacts in a secondary school, and if so, is the boy successful who is adventurous, ambitious, a leader amongst his fellows, and yet has only mediocre scholastic success ? Even if agreement were reached as to the nature of the criterion there would still be doubt as to the accuracy of measurement of its various components, and still greater doubt as to the extent to which each component should be weighted.

In the present inquiry it appears that the head teachers assessed the pupils mainly on the marks obtained in all subjects of the school curriculum. The correlation between the assessments after two and three years, viz. about 0·90 (see p. 22), shows that the basis of assessment *within one school* was much the same at the two times. But it does not follow that all schools adopted the same basis, and in all probability a considerably lower correlation would have resulted if it had been possible for the second assessment to have been made by a head teacher from a different school; one might even doubt whether a correlation of 0·90 would be obtained between assessments of children in one school made at the same time by different teachers. This lack of definition of the criterion places an upper limit to the prognostic value of any admission examination, for the correlation of an examination with future performance can never exceed the square root of the reliability coefficient of the criterion. Professor C. Spearman[1] has stated that correlations between present and future performances up to 0·99 may be expected; this estimate on the above grounds alone appears to be most highly improbable, not to say impossible.

Even if the criterion could be rigorously defined and measured, there are many influences which militate against the successful prediction of children's performance some years later. After children leave the primary schools new interests and abilities develop, temperamental qualities change, and in the different educational and social environment of the secondary school an entirely new attitude towards school work may show itself. There are also vagaries inherent in the admission examination itself; a child's form on the day, special coaching, facilities for home work and encouragement from parents, are but a few of the uncontrollable factors which cause erratic results and reduce accuracy of prediction. It is thus perhaps a matter for wonderment that an examination occupying less than three hours should predict as successfully as it does.

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[1] Report on "Conference on Examinations," III, held at Dinard, France. Bureau of Publications, Teachers' College, Columbia University, New York, 1939. Pages 160 and 185.

Evidence is accumulating that an assessment by teachers in the primary schools gives at least as good a prediction as any of the papers in an admission examination; and it is to be hoped that such assessments will be given more and more consideration in the near future, once it has been found possible to co-ordinate the assessments from different schools. A knowledge of a child's work and abilities extending over several years must necessarily have a large measure of prognostic value, and this valuable source of information should not be neglected.

Turning now to the examination which has been our special study, we have seen that the intelligence test has shown itself significantly superior to the English or Arithmetic papers. This result was achieved in spite of the fact that the two intelligence tests in question performed their function only passably well. The discriminatory efficiency of the tests was low, as evidenced by the standard deviations of the raw scores of the candidates, 11·58 and 13·54 respectively. At the present time tests could readily be constructed to give standard deviations of from 20 to 22. Such increased discrimination would be reflected in higher reliability coefficients, proportionately higher correlations with all other variables, and higher regression coefficients.

With an improved intelligence test the multiple correlation of the three variables with the criterion would be appreciably higher than that now found, viz. 0·764. The present multiple correlation is, however, no mean achievement, for it is an underestimate of the true correlation, since the criterion is but loosely defined and inaccurately measured. Professor Thorndike at the International Conference on Examinations[1] held at Dinard in 1938 remarked in relation to another examination: 'I am convinced that no practical wisdom of any teacher would ever give you a prophecy, that is as good as this ·80 that you have already. . . . I very much doubt whether at the present time a year under observation in an orientation school will give you a correlation for the future of much better than 0·80."

12. SUMMARY

1. The head teachers' assessments of 765 children at the end of their second and third year in secondary schools have been compared with their scores in an admission examination comprising a group verbal intelligence test, an English paper and an Arithmetic paper.

2. The statistics relating to this selected sample of secondary-school children were converted into estimates for the complete unselected group of examination age, and from these data comparative measures of the effectiveness of the three parts of the examination were obtained.

3. The multiple correlation of Intelligence, English and Arithmetic, with the assessment after two years at

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[1] *Loc. cit.*, page 185.

secondary school was 0·743; with assessment after three years it was 0·764. These correlations are depressed below their true values because the criterion could not be exactly defined nor accurately measured.

4. The best *single* measure of later performance is given by the intelligence test; its superiority to the English and Arithmetic papers is almost significant at the 5 per cent. point.

5. When the three variables are considered *as a team*, the intelligence test is significantly superior in prognostic value to the English or Arithmetic papers: it is, in fact, almost equal to the English and Arithmetic papers taken together in their best weighted combination.

6. A still greater predictive value of the intelligence test could be expected with a test specially designed for the age and ability of the candidates for admission to secondary schools; a corresponding increase in the multiple correlation of the team of tests with the criterion would follow.

7. The prediction of assessments made in the secondary school after three years was slightly better than that of assessments after two years. The difference, however, was far from significant.

8. Some comments on statistical methods have been made. It is pointed out that regression coefficients must be related to standard scores before they become measures of the relative merits of the predicting variables. Further, if in the sample of children the degree of selection is not the same for each independent variable, the regression coefficients must be corrected for the influence of selection. In practice it appears that moderate differences in the degree of selection in the several independent variables do not affect seriously the relative values of the regression coefficients.

9. The size of the regression coefficients and correlation coefficients is adventitiously determined by the rigour of selection of the secondary-school sample. It is therefore necessary to relate all such statistics to the complete population of children about to leave the elementary schools. In order to do so the standard deviations and the intercorrelations of the predicting variables in the population must be known, and thus the multiple correlation in the population may be found.

APPENDIX I

THE CALCULATION OF REGRESSION COEFFICIENTS AND THEIR STANDARD ERRORS

Regression Coefficients, $b_{12\cdot 34} \ldots$, etc. (for standard scores), of a criterion, x_1, on several experimentally independent variables, x_2, x_3, x_4, . . . , are given by the following equations :

$$b_{12\cdot 34} \ldots = r_{12}r^{22} + r_{13}r^{23} + r_{14}r^{24} + \ldots$$
$$b_{13\cdot 24} \ldots = r_{12}r^{23} + r_{13}r^{33} + r_{14}r^{34} + \ldots \text{ etc.,}$$

where r^{ij} is an element of the reciprocal matrix of the correlations of the *independent* variables.

The Multiple Correlation $r_{1\cdot 234} \ldots$, is given by

$$r^2_{1\cdot 234} \ldots = b_{12\cdot 34} \ldots r_{12} + b_{13\cdot 24} \ldots r_{13} + b_{14\cdot 23} \ldots r_{14} + \ldots$$

The Standard Errors of the Regression Coefficients are given by

$$\sigma^2_{b_{12\cdot 34} \ldots} \quad \frac{r^{22}(1 - r^2_{1\cdot 234} \ldots)}{N - p - 1}$$

$$\sigma^2_{b_{13\cdot 24} \ldots} = \frac{r^{33}(1 - r^2_{1\cdot 234} \ldots)}{N - p - 1}, \text{ etc.,}$$

where p is the number of independent variables, and N the number of cases on which the correlation coefficients are based.

The Standard Error of the Difference between Two Regression Coefficients is given by

$$\sigma^2_{(b_{12} - b_{13})} = \frac{(r^{22} + r^{33} - 2r^{23})(1 - r^2_{1\cdot 234} \ldots)}{N - p - 1}$$

It is well to remark that r^{23} is itself usually negative in sign, so that the term $(- 2r^{23})$ then becomes positive.

In the course of our inquiry we found two sets of regression coefficients, one set (b_{1i}) for a selected sample of children and the other (B_{1i}) an estimated set for the unselected population. The standard errors of the second set are readily found from those of the first set as follows:

Putting $B_{1i}/b_{1i} = k_{1i}$, then $\sigma_{B_{1i}} = k_{1i}\sigma_{b_{1i}}$; thus, $\sigma_{B_{12}} = k_{12}\sigma_{b12} = B_{12}\sigma_{b_{12}}/b_{12}$. Also

$$\sigma^2_{(B_{12}-B_{13})} = \frac{(k^2_{12}r^{22} + k^2_{13}r^{33} - 2k_{12}k_{13}r^{23})\,(1 - r^2_{1\cdot 234\,\ldots})}{N - p - 1};$$

similarly for $\sigma^2_{(B_{12}-B_{14})}$, etc.

The only arithmetical process that requires explanation is the calculation of the reciprocal of the matrix of the correlations of the independent variables with unities as diagonal elements. When there are only three independent variables, the reciprocal matrix is:

$$\begin{bmatrix} (1 - r^2_{34})/\Delta & (r_{24}r_{34} - r_{23})/\Delta & (r_{23}r_{34} - r_{24})/\Delta \\ (r_{24}r_{34} - r_{23})/\Delta & (1 - r^2_{24})/\Delta & (r_{23}r_{24} - r_{34})/\Delta \\ (r_{23}r_{34} - r_{24})/\Delta & (r_{23}r_{24} - r_{34})/\Delta & (1 - r^2_{23})/\Delta \end{bmatrix}$$

where $\Delta = 1 - r^2_{23} - r^2_{24} - r^2_{34} + 2r_{23}r_{24}r_{34}$.

For example $(1 - r^2_{34})/\Delta$ is the element r^{22}, and $(r_{23}r_{24} - r_{34})/\Delta$ is the element r^{34}.

When there are more than three independent variables Aitken's method of pivotal condensation should be used to evaluate the reciprocal matrix. This is fully described by Professor Godfrey H. Thomson in the *Journal of Educational Psychology* (1936, **27**, 44) and an illustrative computation is given in Appendix 3 of this article.

It is scarcely possible to go wrong in the arithmetical work as checks are available at every stage. The following checks are important:

(1) On the reciprocal matrix

$$r^{22} + r_{23}r^{23} + r_{24}r^{24} + \ldots = 1$$
$$r_{23}r^{22} + r^{23} + r_{34}r^{24} + \ldots = 0$$
$$r_{24}r^{22} + r_{34}r^{23} + r^{24} + \ldots = 0, \text{ etc.}$$

In words, the inner product of a row of the original matrix with the corresponding column of its reciprocal is unity, and the inner product of a row with a non-corresponding column is zero.

(2) On the regression coefficients

$$\begin{aligned}
b_{12} \quad &+ b_{13}r_{23} + b_{14}r_{24} + \cdots = r_{12} \\
b_{12}r_{23} &+ b_{13} \quad + b_{14}r_{34} + \cdots = r_{13} \\
b_{12}r_{24} &+ b_{13}r_{34} + b_{14} \quad + \cdots = r_{14}
\end{aligned}$$

APPENDIX II

THE INFLUENCE OF SELECTION ON THE VARIANCE AND COVARIANCES OF THE CRITERION

In our particular problem we know the variances (σ_i^2) and covariances ($\sigma_i\sigma_j r_{ij}$) of the three independent variables in both the secondary-school sample and in the population; we also know the variance and covariances of the criterion in the sample. We require to find the variance and covariances of the criterion in the population, and hence the correlation coefficients between criterion and independent variables in the population.

We use the same notation as previously:

x_1, the criterion or assessment in secondary school;
x_2, I.Q.;
x_3, English score;
x_4, Arithmetic score.

σ and Σ are standard deviations in sample and population respectively, r and R correlation coefficients in sample and population.

Our observed data are as follows. First the matrix of variances and covariances in the sample.

$$= \left[\begin{array}{c:ccc} \sigma_1^2 & \sigma_1\sigma_2 r_{12} & \sigma_1\sigma_3 r_{13} & \sigma_1\sigma_4 r_{14} \\ \hdashline \sigma_1\sigma_2 r_{12} & \sigma_2^2 & \sigma_2\sigma_3 r_{23} & \sigma_2\sigma_4 r_{24} \\ \sigma_1\sigma_3 r_{13} & \sigma_2\sigma_3 r_{23} & \sigma_3^2 & \sigma_3\sigma_4 r_{34} \\ \sigma_1\sigma_4 r_{14} & \sigma_2\sigma_4 r_{24} & \sigma_3\sigma_4 r_{34} & \sigma_4^2 \end{array}\right] = \left[\begin{array}{c:c} U_{qq} & U_{qp} \\ \hdashline U_{pq} & U_{pp} \end{array}\right] \text{ say.}$$

Also the variances and covariances, in the population, of the three variables x_2, x_3 and x_4 through which selection has occurred. These are given in the bottom right-hand quadrant of the partitioned matrix V below.

$$V=\left[\begin{array}{c|ccc} \Sigma_1{}^2 & \Sigma_1\Sigma_2R_{12} & \Sigma_1\Sigma_3R_{13} & \Sigma_1\Sigma_4R_{14} \\ \hline \Sigma_1\Sigma_2R_{12} & \Sigma_2{}^2 & \Sigma_2\Sigma_3R_{23} & \Sigma_2\Sigma_4R_{24} \\ \Sigma_1\Sigma_3R_{13} & \Sigma_2\Sigma_3R_{23} & \Sigma_3 & \Sigma_3\Sigma_4R_{34} \\ \Sigma_1\Sigma_4R_{14} & \Sigma_2\Sigma_4R_{24} & \Sigma_3\Sigma_4R_{34} & \Sigma_4{}^2 \end{array}\right]=\left[\begin{array}{c|c} V_{qq} & V_{qp} \\ \hline V_{pq} & V_{pp} \end{array}\right]\text{say.}$$

It is required to find the values of the elements in the first row or column of V. Partitioning the matrix as shown, we have to find the values of the elements in submatrices V_{qp} or V_{pq} and V_{qq}.

Aitken's formula gives $V_{pq} = V_{pp}C$

and $V_{qq} = D + C'V_{pp}C$,

where $C = U_{pp}{}^{-1}\, U_{pq}$

and $D = U_{qq} - U_{qp}C$, C' being the transpose of C.

It may be noted that the elements of matrix C are the regression coefficients for raw scores.

Although these formulæ may seem forbidding, the computations are within the scope of anyone who can do simple arithmetic accurately. We illustrate from the data for assessments after three years at the secondary school.

For the sample we have $\sigma_1 = 11{\cdot}233$ $r_{12} = {\cdot}454$ $r_{23} = {\cdot}437$

$\sigma_2 = 9{\cdot}67$ $r_{13} = {\cdot}335$ $r_{24} = {\cdot}357$

$\sigma_3 = 12{\cdot}77$ $r_{14} = {\cdot}330$ $r_{34} = {\cdot}194$

$\sigma_4 = 16{\cdot}01$

And for the population,

$\Sigma_2 = 15{\cdot}00$ $R_{23} = {\cdot}833$

$\Sigma_3 = 23{\cdot}44$ $R_{24} = {\cdot}809$

$\Sigma_4 = 25{\cdot}89$ $R_{34} = {\cdot}769$

Substituting in the submatrices of U,

$$U_{qq} = [126{\cdot}180] \quad U_{qp} = [\,49{\cdot}315 \quad 48{\cdot}054 \quad 59{\cdot}347\,]$$

$$U_{pq} = \begin{bmatrix} 49{\cdot}315 \\ 48{\cdot}054 \\ 59{\cdot}347 \end{bmatrix} \quad U_{pp} = \begin{bmatrix} 93{\cdot}509 & 53{\cdot}963 & 55{\cdot}270 \\ 53{\cdot}963 & 163{\cdot}073 & 39{\cdot}663 \\ 55{\cdot}270 & 39{\cdot}663 & 256{\cdot}320 \end{bmatrix}$$

We now have to find U_{pp}^{-1}. We shall denote the elements in U_{pp} by u_{22}, u_{23}, u_{24}, etc. ; thus $u_{22} = 93{\cdot}509$, $u_{34} = 39{\cdot}663$; and the corresponding elements in the reciprocal matrix by u^{22}, u^{23}, u^{24}, etc.

If there are more than three independent variables the reciprocal of U_{pp} is best found by Aitken's method of pivotal condensation given in Appendix 3. In the present instance we may proceed by a shorter method. Since in U_{pp} the diagonal elements are not unity, the short method for three variables given in Appendix 1 is not applicable here, but the principle involved is the same. We find first the determinant Δ of the matrix U_{pp}. This is given by

$$\Delta = u_{22}(u_{33}u_{44} - u_{34}^2) + u_{23}(u_{24}u_{34} - u_{23}u_{44}) + u_{24}(u_{23}u_{34} - u_{24}u_{33}) = 2{,}753{,}503{\cdot}106$$

Then U_{pp}^{-1}

$$= \begin{bmatrix} (u_{33}u_{44} - u_{34}^2)/\Delta & (u_{24}u_{34} - u_{23}u_{44})/\Delta & (u_{23}u_{34} - u_{24}u_{33})/\Delta \\ (u_{24}u_{34} - u_{23}u_{44})/\Delta & (u_{22}u_{44} - u_{24}^2)/\Delta & (u_{24}u_{23} - u_{22}u_{34})/\Delta \\ (u_{23}u_{34} - u_{24}u_{33})/\Delta & (u_{24}u_{23} - u_{22}u_{34})/\Delta & (u_{22}u_{33} - u_{23}^2)/\Delta \end{bmatrix}$$

$$= \begin{bmatrix} 146{\cdot}089 & -42{\cdot}272 & -24{\cdot}960 \\ -42{\cdot}272 & -75{\cdot}952 & -2{\cdot}638 \\ -24{\cdot}960 & -2{\cdot}638 & 44{\cdot}804 \end{bmatrix} \times 10^{-4},$$

the factor 10^{-4} being applied to each element in the matrix.

We may check this result by noting whether the inner product of each row of U_{pp} with the corresponding column (or row since the matrix is symmetrical) of U_{pp}^{-1} is unity ; also whether the inner product of each row of U_{pp} with a non-corresponding column (or row) of U_{pp}^{-1} is zero (see Appendix 1). The inner product is the sum of the products of corresponding elements.

For example,

$$u_{23}u^{23} + u_{33}u^{33} + u_{34}u^{34}$$
$$= (-\,53{\cdot}963 \times 42{\cdot}272 + 163{\cdot}073 \times 75{\cdot}952 - 39{\cdot}663 \times 2{\cdot}638) \times 10^{-4} = 1{\cdot}000$$

$$u_{23}u^{24} + u_{33}u^{34} + u_{34}u^{44}$$
$$= (-\,53{\cdot}963 \times 24{\cdot}960 - 163{\cdot}073 \times 2{\cdot}638 + 39{\cdot}663 \times 44{\cdot}804) \times 10^{-4} = 0{\cdot}000$$

We next find $C = U_{pp}^{-1}U_{pq} = \begin{bmatrix} \cdot 369174 \\ \cdot 140860 \\ \cdot 130131 \end{bmatrix}$. These figures are obtained by forming the inner product of successive rows of U_{pp}^{-1} with U_{pq}. Thus

$$\cdot 369174 = (146{\cdot}089 \times 49{\cdot}315 - 42{\cdot}272 \times 48{\cdot}054 - 24{\cdot}960 \times 59{\cdot}347) \times 10^{-4}$$

and $\cdot 140860 = (-42{\cdot}272 \times 49{\cdot}315 + 75{\cdot}952 \times 48{\cdot}054 - 2{\cdot}638 \times 59{\cdot}347) \times 10^{-4}$.

The elements in C are the regression coefficients for raw scores, w_{12}, w_{13} and w_{14}, referred to on pages 29 and 34. There we noted that $w_{12} = b_{12} \times \frac{\sigma_1}{\sigma_2} = {\cdot}3178 \times \frac{11{\cdot}233}{9{\cdot}67} = \cdot 36917$ agreeing with the value in C.

The next move is to find $D = U_{qq} - U_{qp}C$.

$$U_{qp}C = [49{\cdot}315 \quad 48{\cdot}054 \quad 59{\cdot}347] \times \begin{bmatrix} \cdot 369174 \\ \cdot 140860 \\ \cdot 130131 \end{bmatrix}$$

Forming the inner product of row and column as before, $U_{qp}C = 32{\cdot}698$, whence $D = 126{\cdot}180 - 32{\cdot}698 = 93{\cdot}482$.

Finally we have to evaluate $C'V_{pp}C$. The values of all the elements in V_{pp} are known, viz.:

$$V_{pp} = \begin{bmatrix} 225{\cdot}00 & 292{\cdot}883 & 314{\cdot}175 \\ 292{\cdot}883 & 549{\cdot}434 & 466{\cdot}677 \\ 314{\cdot}175 & 466{\cdot}677 & 670{\cdot}292 \end{bmatrix}$$

For example, $292{\cdot}883 = \Sigma_2\Sigma_3R_{23}$, and $670{\cdot}292 = \Sigma_4^2$

Forming the inner product of successive rows of V_{pp} with C.

we have $V_{pp}C = \begin{bmatrix} 165{\cdot}204 \\ 246{\cdot}247 \\ 268{\cdot}947 \end{bmatrix}$ Thus $165{\cdot}204 = 2{,}25{\cdot}000 \times \cdot 369174 + 292{\cdot}883 \times \cdot 140860 + 314{\cdot}175 \times \cdot 130131$.

$$\text{Then } C'(V_{pp}C) = [\cdot 369174 \quad \cdot 140860 \quad \cdot 130131] \times \begin{bmatrix} 165\cdot 204 \\ 246\cdot 247 \\ 268\cdot 947 \end{bmatrix}$$

$$= 130\cdot 674$$

$$\text{and } D + C'V_{pp}C \; (= V_{qq}) = 93\cdot 482 + 130\cdot 674 = 224\cdot 156.$$

We are now in a position to give values to all elements in the variance matrix, V, for the population. The matrix is

$$V = \begin{array}{c|c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} & \\ \begin{bmatrix} 224\cdot 156 & 165\cdot 204 & 246\cdot 247 & 268\cdot 947 \\ 165\cdot 204 & 225\cdot 000 & 292\cdot 883 & 314\cdot 175 \\ 246\cdot 247 & 292\cdot 883 & 549\cdot 434 & 466\cdot 677 \\ 268\cdot 947 & 314\cdot 175 & 466\cdot 677 & 670\cdot 292 \end{bmatrix} & \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \end{array}$$

We have immediately, element $v_{11} = \Sigma_1{}^2 = 224\cdot 156$ or $\Sigma_1 = 14\cdot 972$.

$$\text{Also } R_{12} = \frac{v_{12}}{\sqrt{v_{11}v_{22}}} = \left(\frac{\Sigma_1\Sigma_2R_{12}}{\sqrt{\Sigma^2{}_1 \cdot \Sigma^2{}_2}}\right) = \frac{165\cdot 204}{\sqrt{224\cdot 156 \times 225\cdot 000}}$$

$$= \cdot 7356$$

$$\text{Similarly } R_{13} = \cdot 7017$$

$$R_{14} = \cdot 6938$$

Regression coefficients for standard scores applying to the population are obtained from w_{12}, w_{13} and w_{14}, the elements in matrix C, thus :

$$B_{12} = w_{12} \times \frac{\Sigma_2}{\Sigma_1} = \cdot 369174 \times \frac{15\cdot 00}{14\cdot 972} = \cdot 3699$$

$$B_{13} = w_{13} \times \frac{\Sigma_3}{\Sigma_1} = \cdot 140860 \times \frac{23\cdot 44}{14\cdot 972} = \cdot 2205$$

$$B_{14} = w_{14} \times \frac{\Sigma_4}{\Sigma_1} = \cdot 130131 \times \frac{25\cdot 89}{14\cdot 972} = \cdot 2250$$

These regression coefficients are checked by the method given at the end of Appendix 1 : e.g. :

$B_{12} + B_{13}R_{23} + B_{14}R_{24} = \cdot 3699 + \cdot 2205 \times \cdot 833 + \cdot 2250 \times \cdot 809 = \cdot 7356$, agreeing with the ascertained value of R_{12}.

The Multiple Correlation of the criterion with the three independent variables in the population is given by

$R^2_{1\cdot 234} = \cdot 3699 \times \cdot 7356 + \cdot 2205 \times \cdot 7017 + \cdot 2250 \times \cdot 6938 = \cdot 582928$, whence $R_{1\cdot 234} = \cdot 7635$.

APPENDIX III

THE CALCULATION OF THE RECIPROCAL OF A MATRIX BY PIVOTAL CONDENSATION (AITKEN'S METHOD)

We demonstrate with the matrix U_{pp} in Appendix 2. As this matrix is of order 3 × 3, that is, has but three columns and three rows, it is much shorter to find its reciprocal by the method given in Appendix 2 ; the method of pivotal condensation, however, is more convenient for matrices of higher order.

The working is now given (p. 56), and the explanation of the method follows.

Explanation of Working

1. The matrix is first written down as shown to the left of rows 1, 3 and 4. Row 2 is for the moment left blank. The matrix is bordered by three matrices ; immediately below by a matrix of the same order (in our case 3 × 3) with unity in each diagonal position and zero elsewhere (the dots represent zero) ; to the right by a matrix of the same order with — 1 in each diagonal position and zero elsewhere ; to the bottom right by a matrix of the same order with zero elements throughout. In the check column is entered the sum of the elements in each row.

2. The top left figure of the matrix, 93·509, is called the pivot. The first move is to divide all elements in row 1 by the pivot ; this is more easily done by multiplying each by the reciprocal of the pivot, which is shown as ·0106942 to the left of the matrix. The products of the reciprocal are written in row 2 which was left blank for this purpose. (When the reciprocal of a matrix of correlation coefficients with unities as diagonal elements has to be found, this process is, of course, omitted since the pivot itself is unity.) The sum of the elements in row 2, 2·157465, is then written in the check column, and checked against 201·742 × ·0106942.

3. The process of condensation now begins. The first row

							Check Column	Row No.	
÷ 93·509 (recip. = ·0106942)	93·509	53·963	55·270	−1	·	·	201·742	1	Slab 1
	1	·577091	·591068	− ·010694	·	·	2·157465	2	
	53·963	163·073	39·663	·	−1	·	255·699	3	
	55·270	39·663	256·320	·	·	−1	350·253	4	
	1	·	·	·	·	·	1·000	5	
	·	1	·	·	·	·	1·000	6	
	·	·	1	·	·	·	1·000	7	
÷ 131·931 (recipr. = ·00757972)		131·931	7·7672	·57708	−1	·	139·275	8	Slab 2
		1	·058873	·0043741	− ·0075797	·	1·055666	9	
		7·7672	223·652	·59106	·	− 1	231·010	10	
		− ·577091	− ·591068	·010694	·	·	−1·157465	11	
		1	·	·	·	·	1·000	12	
		·	1	·	·	·	1·000	13	
÷ 223·195 (recipr. = ·00448039)			223·195	·55709	·058873	− 1	222·811	14	Slab 3
			1	·0024960	·00026377	− ·0044804	·998279	15	
			− ·557093	·0132183	− ·0043741	·	− ·548249	16	
			− ·058873	− ·0043741	·0075797	·	− ·055667		
			1	·	·	·	1·000		
				·014609	− ·0042272	− ·0024960	·0078858		Slab 4
				− ·0042272	·0075952	− ·0002638	·0031042		
				− ·0024960	− ·0002638	·0044804	·001721		

of Slab 2, that is row 8, is obtained from rows 2 and 3 of Slab 1 as follows:

$$\begin{aligned} 131{\cdot}931 &= 1 \times 163{\cdot}073 - 53{\cdot}963 \times {\cdot}577091 \\ 7{\cdot}7672 &= 1 \times 39{\cdot}663 - 53{\cdot}963 \times {\cdot}591068 \\ {\cdot}57708 &= 1 \times 0 - 53{\cdot}963 \times -{\cdot}010694, \text{ and so on.} \end{aligned}$$

The entry in the check column of row 8 is obtained in the same way, viz. 139·275 = 1 × 255·699 − 53·963 × 2·157465. The sum of the elements to the left of the check column must be equal to 139·275 if the arithmetic is correct. Error of a few units in the last significant figure is, of course, permissible when arithmetical approximations have been made. In such cases it is advisable to write in the check column the sum of the elements in the row rather than the difference of products.

4. Row 9 may now be completed by dividing each of the elements in row 8 by the pivot 131·931, or multiplying by ·00757972.

The remaining rows of Slab 2 are found from pairs of rows in Slab 1 as was row 8. Row 10, for example, is derived from rows 2 and 4, row 11 from rows 2 and 5, etc. Thus, 223·652 in row 10 is obtained from 1 × 256·320 − 55·270 × ·591068. Note that two elements in row 2 always figure in the calculation for each element of Slab 2.

5. When Slab 2 is completed, the elements of Slab 3 are computed from Slab 2 in the same way, row 14 being derived from rows 9 and 10, row 16 from rows 9 and 11, and so on. Thus, ·0132183 in row 16 is derived from 1 × ·010694 − (− ·577091 × ·0043741).

6. The process of condensation is continued until no figures remain in the left-hand block under the original matrix. The 3 × 3 matrix to the left of the check column of Slab 4 is the desired reciprocal matrix. It is the same as found by the shorter method in Appendix 2.

7. The number of condensations to be made will be equal to the number of columns in the original matrix. The condensation of a matrix of correlation coefficients is of frequent occurrence for the calculation of regression coefficients; it is somewhat less tedious than that of the present

matrix of covariances since the position of the decimal points is easier to locate. In general it is advisable to retain from four to six significant figures according to the accuracy of the elements in the original matrix.

8. Although each row of the computation is checked by the figure in the check column, a further check can be made as described at the end of Appendix I.

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